

Rosen, Discrete Mathematics and Its Applications, 6th edition
Extra Examples

Section 12.1—Modeling Computation



— Page references correspond to locations of Extra Examples icons in the textbook.

p.790, icon at Example 8

#1. Let $G = (V, T, S, P)$ be a grammar where $V = \{S, A, B, a, b\}$ is the vocabulary and $T = \{a, b\}$ is the set of terminal elements. Determine whether the following set of productions is a:

- (i) a type 0 grammar, but not a type 1 grammar.
- (ii) a type 1 grammar, but not a type 2 grammar.
- (iii) a type 2 grammar, but not a type 3 grammar.

$$S \rightarrow ABA, A \rightarrow bB, B \rightarrow ba.$$

Solution:

It is a type 2 grammar because the left side of each production has a single nonterminal symbol. It is not a type 3 grammar because of the form of the right side of the third production.

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#2. Let $G = (V, T, S, P)$ be a grammar where $V = \{S, A, B, a, b\}$ is the vocabulary and $T = \{a, b\}$ is the set of terminal elements. Determine whether the following set of productions is a:

- (i) a type 0 grammar, but not a type 1 grammar.
- (ii) a type 1 grammar, but not a type 2 grammar.
- (iii) a type 2 grammar, but not a type 3 grammar.

$$S \rightarrow AB, B \rightarrow bAa, bAa \rightarrow a.$$

Solution:

It is automatically a type 0 grammar. It is not a type 1 grammar because the third production is not noncontracting.

p.793, icon at Example 13

#1.

(a) What is the Backus-Naur form of the grammar described as follows:

1. a sentence is made up of a noun phrase followed by a verb phrase or else by a noun phrase followed by a verb phrase followed by a noun phrase.
2. a noun phrase is made up of a noun, an adjective followed by a noun, or an article followed by a noun.
3. a verb phrase is made up of a verb.
4. articles are *a* and *the*.
5. adjectives are *lengthy*, *boring*, and *inaccurate*.

6. nouns are *book*, *newspaper*, and *information*.
7. verbs are *reads* and *contains*.

(b) Explain how “the newspaper contains lengthy information” can be obtained.

Solution:

(a)

$\langle \textit{sentence} \rangle ::= \langle \textit{noun phrase} \rangle \langle \textit{verb phrase} \rangle \mid \langle \textit{noun phrase} \rangle \langle \textit{verb phrase} \rangle \langle \textit{noun phrase} \rangle$

$\langle \textit{noun phrase} \rangle ::= \langle \textit{noun} \rangle \mid \langle \textit{article} \rangle \langle \textit{noun} \rangle \mid \langle \textit{adjective} \rangle \langle \textit{noun} \rangle$

$\langle \textit{verb phrase} \rangle ::= \langle \textit{verb} \rangle$

$\langle \textit{article} \rangle ::= a \mid the$

$\langle \textit{adjective} \rangle ::= lengthy \mid boring \mid inaccurate$

$\langle \textit{noun} \rangle ::= newspaper \mid information$

$\langle \textit{verb} \rangle ::= reads \mid contains$

(b) “the newspaper” is a noun phrase since it has the form $\langle \textit{article} \rangle \langle \textit{noun} \rangle$; “contains” is a verb phrase since it has the form $\langle \textit{verb} \rangle$; “lengthy information” is a noun phrase since it has the form $\langle \textit{adjective} \rangle \langle \textit{noun} \rangle$.

Therefore “the newspaper contains lengthy information” is a sentence since it has the form $\langle \textit{noun phrase} \rangle \langle \textit{verb phrase} \rangle \langle \textit{noun phrase} \rangle$
