Stem-and-Leaf Plots and Mean, Median, and Mode

What *you should learn*

GOAL 1 Make and use a stem-and-leaf plot to put data in order.

GOAL 2 Find the mean, median, and mode of data, such as tire usage in **Example 3**.

Why *you should learn it*

▼ To describe **real-life** data, such as the ages of the residents of Alcan, Alaska, in **Example 1**.

GOAL 1 MAKING STEM-AND-LEAF PLOTS

A **stem-and-leaf plot** is an arrangement of digits that is used to display and order numerical data.

EXAMPLE 1 *Making a Stem-and-Leaf Plot*

The following data show the ages of the 27 residents of Alcan, Alaska. Make a stem-and-leaf plot to display the data. ▶ Source: U.S. Bureau of the Census

45	1	52	42	10	40	50	40	7
46	19	35	3	11	31	6	41	12
43	37	8	41	48	42	55	30	58

SOLUTION

Use the digits in the tens' place for the *stems* and the digits in the ones' place for the *leaves*. The key shows you how to interpret the digits.

Unordered stem-and-leaf plot

Stems

0	1 7 3 6 8
1	0 9 1 2
2	
3	5 1 7 0
4	5 2 0 0 6 1 3 1 8 2
5	2 0 5 8

Leaves

Key: 4|5 = 45

You can order the leaves to make an *ordered* stem-and-leaf plot.

Ordered stem-and-leaf plot

Stems

0	1 3 6 7 8
1	0 1 2 9
2	
3	0 1 5 7
4	0 0 1 1 2 2 3 5 6 8
5	0 2 5 8

Leaves

Key: 4|5 = 45

Your choice of place values for the stem and the leaves will depend on the data. For data between 0 and 100, the leaves are the digits in the ones' place. Be sure to include a key that explains how to interpret the digits.
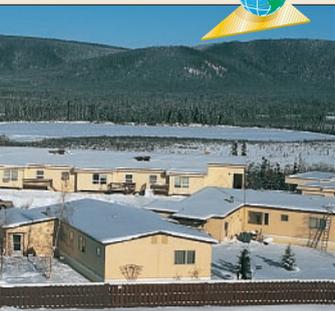
A **measure of central tendency** is a number that is used to represent a typical number in a data set. What value is "most typical" may depend on the type of data, and it may be a matter of opinion. There are three commonly used measures of central tendency of a collection of numbers: the *mean*, the *median*, and the *mode*.

CONCEPT SUMMARY

MEASURES OF CENTRAL TENDENCY

- The **mean,** or **average,** of *n* numbers is the sum of the numbers divided by *n*.

- The **median** of *n* numbers is the middle number when the numbers are written in order. If *n* is even, the median is taken to be the average of the two middle numbers.

- The **mode** of *n* numbers is the number that occurs most frequently. A set of data can have more than one mode or no mode.

REAL LIFE

Population

EXAMPLE 2 *Finding the Mean, Median, and Mode*

Find the measure of central tendency of the ages of the residents of Alcan, Alaska, given in Example 1.

a. mean   **b.** median   **c.** mode

SOLUTION

a. To find the mean, add the 27 ages and divide by 27.

$$\text{mean} = \frac{1 + 3 + \cdots + 55 + 58}{27} \qquad \textbf{Definition of mean}$$

$$= \frac{853}{27} \qquad \textbf{Add ages.}$$

The mean is $\frac{853}{27}$, or about 32.

b. To find the median, write the ages in order and find the middle number. To order the ages, use the ordered stem-and-leaf plot in Example 1.

1	3	6	7	8	10	11	12	19
30	31	35	37	**40**	40	41	41	42
42	43	45	46	48	50	52	55	58

From this list, you can see that the median age is 40. Half of the ages fall below 40 and half of the ages are 40 or older.

**c.** To find the mode, use the ordered list in part (b).

There are three modes, 40, 41, and 42.

Many collections of numbers in real life have graphs that are *bell shaped.* For such collections, the mean, the median, and the mode are about equal.

EXAMPLE 3 *A Bell-Shaped Distribution*

A tire manufacturer conducted a survey of 118 people who had purchased a particular type of tire. Each person was asked to report the number of miles driven before replacing the tires. Responses are shown in the histogram. Find the median and the mode of the data.

Miles Until Tire Replacement
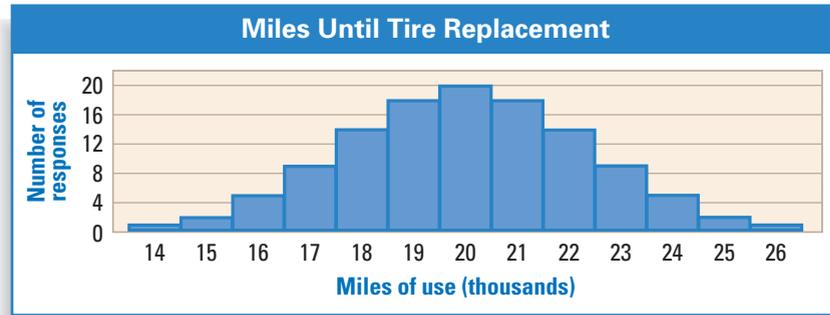
SOLUTION The tallest bar is at 20,000 miles, so that is the mode. The number of responses to the left of that bar is the same as the number of responses to the right, so 20,000 miles is also the median. The mean is also 20,000 miles.

EXAMPLE 4 *Interpreting Measures of Central Tendency*

80-Year-Old Graduates at Elliot School

Elliot School had 28 graduates this year. The town turned out to witness the graduation of John Wilson, age 80. Of the other graduates, seven were 17 years old, nineteen were 18 years old, and one was 19 years old. One person remarked, "The average age of the graduates must be about 35 years."

Was the person's estimate of the average age correct? Would the mean be the most representative measure of central tendency?

SOLUTION

$$\text{Mean} = \frac{7(17) + 19(18) + 1(19) + 1(80)}{28} = \frac{560}{28} = 20$$

The person's estimate was wrong, because the mean is 20.

Because both the median and the mode of the ages are 18 years, the mean is not the most representative measure of central tendency.

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People usually use the mean to represent a typical number in a data set. If some data are very different, the median may be more representative of the data.

GUIDED PRACTICE

Vocabulary Check ✓ **1.** Define the *mean*, the *median*, and the *mode* of a collection of numbers.

Concept Check ✓ **2.** Explain why you might make a stem-and-leaf plot for a data collection.

3. Give an example in which the mean of a collection of numbers is *not* representative of a typical number in the collection.

Skill Check ✓ **4.** Make a stem-and-leaf plot 22, 34, 11, 55, 13, 22, for the data at the right. 30, 21, 39, 48, 38, 46

Find the mean, the median, and the mode of the collection of numbers.

5. 2, 2, 2, 2, 4, 4, 5 **6.** 5, 10, 15, 1, 2, 3, 7, 8

7. −4, 8, 9, −6, −2, 1 **8.** 6.5, 3.2, 1.7, 10.1, 3.2

9. Use the stem-and-leaf plot below to order the data set.

```
1 | 6 3 8 1 7 5 3
2 | 8 2 5 0 7 7 0
3 | 8 3 1 4 7        Key: 3 | 8 = 38
```

10. 🌐 **AGES** If someone said that the mean age of everyone in your algebra class is about $16\frac{1}{2}$ years, do you think the age of the teacher was included in the calculation? Explain.

PRACTICE AND APPLICATIONS

MAKING STEM-AND-LEAF PLOTS **Make a stem-and-leaf plot for the data. Use the result to list the data in increasing order.**

11. 60, 74, 75, 63, 78, 70, 50, 74, 52, 74, 65, 78, 54

12. 24, 29, 17, 50, 39, 51, 19, 22, 40, 45, 20, 18, 23, 30

13. 4, 31, 22, 37, 39, 24, 2, 28, 1, 26, 28, 30, 28, 3, 20, 20, 5

14. 15, 39, 13, 31, 46, 9, 38, 17, 32, 10, 12, 45, 30, 1, 32, 23, 32, 41

15. 87, 61, 54, 77, 79, 86, 30, 76, 52, 44, 48, 76, 87, 68, 82, 61, 84, 33, 39, 68, 37, 80, 62, 81, 76

16. 48, 10, 48, 25, 40, 42, 44, 23, 21, 13, 50, 17, 18, 19, 21, 57, 35, 33, 25, 50, 13, 12, 46

FINDING MEAN, MEDIAN, AND MODE **Find the mean, the median, and the mode of the collection of numbers.**

17. 4, 2, 10, 6, 10, 7, 10 **18.** 1, 2, 1, 2, 1, 3, 3, 4, 3

19. 8, 5, 6, 5, 6, 6 **20.** 4, 4, 4, 4, 4, 4

21. 5, 3, 10, 13, 8, 18, 5, 17, 2, 7, 9, 10, 4, 1

22. 12, 5, 6, 15, 12, 9, 13, 1, 4, 6, 8, 14, 12

23. 3.56, 4.40, 6.25, 1.20, 8.52, 1.20 **24.** 161, 146, 158, 150, 156, 150

🌎 **BASEBALL** In Exercises 25–28, use the following information.

The table shows the number of shutouts that ten baseball pitchers had in their careers. A shutout is a complete game pitched without allowing a run.

25. Find the mean and the median for the set of data.

26. Write the numbers in decreasing order.

27. Does the set of data have a mode? If so, what is it?

28. CRITICAL THINKING What number could you add to the set of data to make it have more than one mode? Explain why you chose the number.

Pitcher	Shutouts
Warren Spahn	63
Christy Mathewson	80
Eddie Plank	69
Nolan Ryan	61
Bert Blyleven	60
Don Sutton	58
Grover Alexander	90
Walter Johnson	110
Cy Young	76
Tom Seaver	61

▶ Source: *The Baseball Encyclopedia*

29. 🌎 **AVERAGE SALARY** A small town has an adult population of 100. One person is a multimillionaire who makes $5,000,000 per year. The mean annual salary in the town is $74,750. Is the mean a fair measure of a typical salary in the town? What is the mean annual salary of the other 99 adults?

30. 🌎 **NUMBER OF HOUSEHOLDS** The U.S. Bureau of the Census reports on the number of households (in thousands) in each state. Some of the data are shown in the table. Is one measure of central tendency the most representative of the typical number of households in these states? Explain your reasoning.

31. Use the data in Exercise 30. If you exclude California from the data, is one measure of central tendency the most representative of the data? Explain your reasoning.

State	Households
Arizona	1,687
California	11,101
Colorado	1,502
Idaho	430
Montana	341
Nevada	619
New Mexico	619
Utah	639
Wyoming	184

INTERNET DATA UPDATE Visit our Web site www.mcdougallittell.com

32. CRITICAL THINKING Create two different sets of data, each having 10 ages. Create one set so that the mean age is 16 and the median age is 18. Create the other set so that the median age is 16 and the mean age is 18.

33. 🌎 **BIRTHDAYS** Use a stem-and-leaf plot (months as stems, days as leaves) to write the birthdays in order from earliest in the year to latest (1 = January, 2 = February, and so on). Include a key with your stem-and-leaf plot.

10–11	4–14	7–31	12–28	4–17
2–22	8–21	1–24	9–12	1–3
4–30	10–17	6–5	1–25	5–10
12–9	4–1	8–26	12–15	3–17
4–30	2–3	11–11	6–13	11–4
6–24	6–3	4–8	2–20	11–28

34. 🌎 **WEIGHTS** The numbers shown below represent the weights (in pounds) of twelve high school students. Use a stem-and-leaf plot to order the weights of the students from least to greatest. (*Hint:* Use 17│7 = 177 for your key.)

177, 164, 108, 158, 195, 206, 106, 126, 143, 118, 148, 146

35. 🌎 **ASTEROIDS** The numbers below are the mean distances from the sun (in millions of miles) of the first ten asteroids. Use a stem-and-leaf plot to order the distances from least to greatest. (*Hint:* Use 25│70 = 257.0 for your key.)

257.0, 257.4, 247.8, 219.3, 239.3, 225.2, 221.4, 204.4, 221.7, 222.6

🌎 **VACATIONS** In Exercises 36–38, use the following information.

A business conducted a survey of its employees who go on vacation every year. Each person was asked to report the number of miles traveled (to the nearest hundred) on the last vacation. The responses are shown in the histogram.
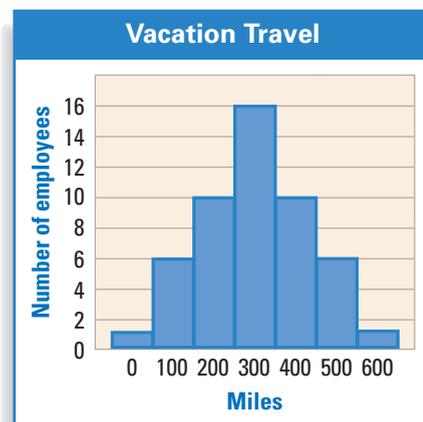
Vacation Travel

36. Find the median of the data.

37. Find the mode of the data.

38. *Writing* Suppose a new employee traveled 400 miles on the last vacation. Decide whether this would affect the median or the mode. Explain your thinking.

EXTENSION: DOUBLE STEM-AND-LEAF PLOT In Exercises 39 and 40, use the following information.

Another type of visual model that you can use is a *double stem-and-leaf plot.* This type of visual model is used to compare two sets of data. The Stem column is in the middle of the plot, so that the "stem" can apply to both sets of data.

The double stem-and-leaf plot below shows the number of all-time tournament wins by the top 17 male and female professional golfers.

▶ Source: Professional Golfers Association

Male	Stem	Female
1	8	8 2
0	7	
3 0	6	
2 1	5	7 5 0
0 0	4	4 2 2
0 1 8 6 3 2 1	3	8 5 2 1 1 1
9 9	2	9 9 6

Key: 1 │ 3 │ 8 = 31, 38

39. What conclusions can be drawn from the double stem-and-leaf plot?

40. Use the data in the double stem-and-leaf plot to make two conventional stem-and-leaf plots. Explain what changes you need to make.

QUANTITATIVE COMPARISON In Exercises 41 and 42, choose the statement below that is true about the given numbers.

Ⓐ The number in column A is greater.

Ⓑ The number in column B is greater.

Ⓒ The two numbers are equal.

Ⓓ The relationship cannot be determined from the given information.

	Column A	Column B
41.	The mean of 1, 2, 3, 4, 5, 5, 10	The mean of 2, 3, 4, 5, 5, 10, 11
42.	The mode of $-2, -2, -2, 4$	The mode of $-2, -2, -3, -3, -3$

★ **Challenge**

43. 🌐 **TEST SCORES** You have test scores of 84, 92, and 76. There will be one more test in the marking period. You want your mean test score for the marking period to be 85 or higher.

   a. Let x represent your last test score. Write an expression for the mean of your test scores for the marking period.

   b. Write and solve an inequality to find what you must score on the last test.

   c. Solve the problem without using algebra. Describe your method. Do your answers agree?

MIXED REVIEW

EVALUATING EXPRESSIONS Evaluate the expression. **(Review 1.3 for 6.7)**

44. $35 + 40 \div 2$ **45.** $120 \times 3 - 2$ **46.** $237 - 188 \div 4$

CHECKING SOLUTIONS Check whether the given number is a solution of the inequality. **(Review 1.4)**

47. $x + 5 > 11; 6$ **48.** $2z - 3 < 4; 4$

49. 🌐 **ROAD TRIP** You and two friends each agree to drive $\frac{1}{3}$ of a 75-mile trip. Which equation would you use to find the number of miles each of you must drive? **(Review 1.5)**

   A. $3x = 75$ **B.** $\frac{1}{3}x = 75$

FINDING SLOPE Find the slope of the graph of the linear function f. **(Review 4.8)**

50. $f(2) = -1, f(5) = 5$ **51.** $f(0) = 5, f(5) = 0$

52. $f(-3) = -12, f(3) = 12$ **53.** $f(0) = 1, f(4) = -1$

SOLVING INEQUALITIES Solve the inequality and graph the solution. **(Review 6.2–6.4)**

54. $-x + 7 > 13$ **55.** $16 - x \le 7$ **56.** $-4 < x + 3 < 7$

57. $7 < 3 - 2x \le 19$ **58.** $|x - 3| < 12$ **59.** $|x + 16| \ge 9$