

Rosen, Discrete Mathematics and Its Applications, 6th edition  
Extra Examples

Section 6.2—Probability Theory



— Page references correspond to locations of Extra Examples icons in the textbook.

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**p.404, icon at Example 3**

#1. You draw 2 cards, one at a time without replacement, at random from a deck of 52 cards. Find

- (a)  $p(\text{second card is a Jack} \mid \text{first card is a Jack})$
- (b)  $p(\text{second card is red} \mid \text{first card is black})$

**Solution:**

- (a) If the first card is a Jack, then there are three Jacks in the remaining deck. Hence the probability that the second card is a Jack is  $3/51 = 1/17$ .
  - (b) If the first card is black, then there are still 26 out of 51 cards that are red. Hence the probability that the second card is red is  $26/51$ .
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**p.405, icon at Example 5**

#1. You write a string of letters of length 3 from the usual alphabet, with no repeated letters allowed. Let  $E_1$  be the event that the string begins with a vowel and  $E_2$  be the event that the string ends with a vowel. Determine whether  $E_1$  and  $E_2$  are independent.

**Solution:**

The sample space has size  $26 \cdot 25 \cdot 24$ . The event  $E_1$  consists of all strings of the form  $\_ \_ \_$ , where the first blank is to be filled in with a vowel. Hence  $|E_1| = 5 \cdot 25 \cdot 24$ . Similarly,  $|E_2| = 25 \cdot 24 \cdot 5$ . Therefore

$$p(E_1) \cdot p(E_2) = \frac{5 \cdot 25 \cdot 24}{26 \cdot 25 \cdot 24} \cdot \frac{25 \cdot 24 \cdot 5}{26 \cdot 25 \cdot 24} = \frac{5}{26} \cdot \frac{5}{26}$$

and

$$p(E_1 \cap E_2) = \frac{5 \cdot 24 \cdot 4}{26 \cdot 25 \cdot 24} = \frac{2}{65}$$

Because  $\frac{5}{26} \cdot \frac{5}{26} \neq \frac{2}{65}$ , the events are not independent

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**p.407, icon at Example 9**

#1. A fair coin is flipped five times. Find the probability of obtaining exactly four heads.

**Solution:**

This is an example of a sequence of five independent Bernoulli trials. In this example, a success is getting heads. The probability of success is  $1/2$  and the probability of failure (getting tails) is  $q = 1 - 1/2 = 1/2$ . Therefore the probability of getting exactly four heads is  $b(4; 5, \frac{1}{2}) = C(5, 4)(\frac{1}{2})^4(1 - \frac{1}{2})^1 \approx 0.156$ .

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**p.407, icon at Example 9**

**#2.** A die is rolled six times in a row. Find

- (a)  $p$ (exactly four 1's are rolled).
- (b)  $p$ (no 6's are rolled).

**Solution:**

(a) This is an example of a sequence of six independent Bernoulli trials, where the probability of success is  $1/6$  and the probability of failure is  $5/6$ . Therefore the probability of rolling exactly four 1's when a die is rolled six times is  $b(4; 6, \frac{1}{6}) = C(6, 4)(\frac{1}{6})^4 (\frac{5}{6})^2 \approx 0.008$ .

(b) In this case a success is "rolling a number other than 6", which has probability  $p = 5/6$  and failure is "rolling a 6", which has probability  $q = 1/6$ . Therefore the probability of rolling no 6's when a die is rolled six times is  $b(6; 6, \frac{5}{6}) = C(6, 6)(\frac{5}{6})^6 (\frac{1}{6})^0 \approx 0.335$ .

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**p.407, icon at Example 9**

**#3.** A quiz consists of 20 true/false questions. You need to have a score of at least 65% in order to pass the quiz. What is the probability that you pass the quiz if you guess at random at each answer?

**Solution:**

This is an example of a sequence of 20 independent Bernoulli trials, where the probability of a success (guessing correctly) and the probability of a failure are both  $1/2$ . To pass, you need to guess correctly on at least 13 of the 20 questions. Therefore, the probability of passing is

$$\begin{aligned} \sum_{i=13}^{20} &= C(20, i) \left(\frac{1}{2}\right)^i \left(1 - \frac{1}{2}\right)^{20-i} \\ &= \sum_{i=13}^{20} C(20, i) \left(\frac{1}{2}\right)^{20} \\ &= \left(\frac{1}{2}\right)^{20} (C(20, 13) + C(20, 14) + \dots + C(20, 20)) \\ &= \left(\frac{1}{2}\right)^{20} (77520 + 38760 + 15504 + 4845 + 1140 + 190 + 20 + 1) \\ &= \left(\frac{1}{2}\right)^{20} (137980) \approx 0.13. \end{aligned}$$

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