

Rosen, Discrete Mathematics and Its Applications, 6th edition
Extra Examples

Section 6.1—An Introduction to Discrete Probability

 — Page references correspond to locations of Extra Examples icons in the textbook.

p.394, icon at Example 1

#1. A computer password consists of five lower case letters, with repeated letters allowed. Find $p(E)$ where E is the event that the password begins with c.

Solution:

The sample space S has 26^5 elements, corresponding to the number of ways to fill in the following five blanks with lower case letters: $_ _ _ _ _$. The event E has 26^4 elements, corresponding to the number of ways to fill in the four blanks in $\underline{c} _ _ _ _$. Therefore

$$p(E) = p(\text{password begins with c}) = \frac{|E|}{|S|} = \frac{26^4}{26^5} = \frac{1}{26} \approx 0.038.$$

Note: In place of writing $p(E)$, we often replace E with the English description of E .

p.394, icon at Example 1

#2. A computer password consists of five lower case letters, with repeated letters allowed.

- (a) Find $p(F_1)$, where F_1 is the event that the password contains no vowels.
- (b) Find $p(F_2)$, where F_2 is the event that the password contains only vowels.

Solution:

(a) The set F_1 has 21^5 elements because each of the five blanks must be filled in with one of 21 consonants. Therefore

$$p(F_1) = p(\text{password contains no vowels}) = \frac{|F_1|}{|S|} = \frac{21^5}{26^5} \approx 0.344.$$

(b) The set F_2 has 5^5 elements because each of the five blanks must be filled in with one of five vowels. Therefore

$$p(F_2) = p(\text{password contains only vowels}) = \frac{|F_2|}{|S|} = \frac{5^5}{26^5} \approx 0.000263.$$

p.394, icon at Example 1

#3. A professor teaches two sections of a calculus course and gave a quiz to the students in each section. In Section 1, 8 students out of 35 got a score of 90 or higher. In Section 2, 11 students out of 28 got a score of 90 or higher.

Find the probability that the student:

- (a) is in Section 1, if the student is chosen at random from among all 63 students.

- (b) is not in Section 1, if the student is chosen at random from among all 63 students.
- (c) scored at least 90 on the quiz, if the student is chosen at random from those in Section 1.
- (d) is in Section 1 and scored at least 90 on the quiz, if the student is chosen at random.
- (e) is in Section 1, if the student is chosen at random from those who scored at least 90 on the quiz.

Solution:

The given information can be displayed in the following table:

	Section 1	Section 2
score ≥ 90	8	11
score < 90	27	17

- (a) Of the 63 students, 35 are in Section 1. Therefore, the probability that the student is in Section 1 is $35/63$.
- (b) The probability that a randomly chosen student is not in Section 1 is equal to the probability that the student is in Section 2, which is $28/63$.
- (c) There are 35 students in Section 1. The probability that the student scored at least 90 is $8/35$.
- (d) Eight students are in Section 1 and scored at least 90 on the quiz. Therefore, the probability that the student is in Section 1 and scored at least 90 on the quiz is $8/63$.
- (e) The number of students who scored at least 90 on the quiz is 19, and, of these, 8 come from Section 1. Therefore, the probability that a randomly chosen student comes from Section 1 is $8/19$.

p.394, icon at Example 1

#4. You flip a coin twice. Find the following:

- (a) $p(E)$ where E is the event of getting heads on the first flip and tails on the second flip.
- (b) $p(F)$ where F is the event of getting one head and one tail in the two flips.

Solution:

The sample space for this experiment is $S = \{HH, HT, TH, TT\}$ where H stands for heads, T for tails, the first letter in each pair is the result of the first flip, and the second letter is the result of the second flip.

- (a) The event $E = \{HT\}$. Thus $p(E) = |E|/|S| = 1/4$.
- (b) The event $F = \{HT, TH\}$. Thus $p(F) = |E|/|F| = 2/4 = 1/2$.

p.394, icon at Example 1

#5. A tetrahedral die is a regular polyhedron consisting of 4 equilateral triangles, with the four faces numbered 1, 2, 3, 4. You roll the pair of tetrahedral dice. Find the probability that the sum is: (a) 2, (b) 3, (c) 4, (d) 5, (e) 6, (f) 7, (g) 8.

Solution:

The sample space consists of the 16 elements:

$$(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4).$$

Thus, the probabilities are: (a) $1/16$, (b) $2/16$, (c) $3/16$, (d) $4/16$, (e) $3/16$, (f) $2/16$, (g) $1/16$.

p.394, icon at Example 1

#6. You pick five numbers, without replacement, from the set $\{1, 2, 3, \dots, 24, 25\}$. What is the probability that the product of the numbers chosen is odd?

Solution:

We can think of the experiment of choosing five numbers in two ways: pick all five numbers at once (i.e., order does not matter), or pick the five numbers in succession (i.e., order matters).

Suppose we interpret the problem as one where we choose the five numbers all at once. Then the sample space S consists of all subsets of five numbers chosen from the set $\{1, 2, 3, \dots, 24, 25\}$, and we have $|S| = C(25, 5)$.

The product will be odd if and only if each of the five chosen numbers is odd. (If any of the five chosen numbers is even, the product will be even.) Therefore

$$p(\text{product is odd}) = \frac{C(13, 5)}{C(25, 5)} = \frac{13!/(5! \cdot 8!)}{25!/(5! \cdot 20!)} = \frac{13! \cdot 20!}{25! \cdot 8!} \approx 0.024.$$

Now suppose that we assume that order does matter. Because the five numbers chosen must all come from the 13 odd numbers between 1 and 25, the number of successes is $P(13, 5)$. The number of possibilities is $P(25, 5)$. Therefore,

$$p(\text{product is odd}) = \frac{P(13, 5)}{P(25, 5)} = \frac{13!/8!}{25!/20!} = \frac{13! \cdot 20!}{25! \cdot 8!} \approx 0.024.$$

In this case the same answer is obtained whether we assume order matters or that order does not matter.

p.394, icon at Example 1

#7. Suppose $S = \{1, 2, \dots, 20\}$. You select a subset $T \subseteq S$ of size three. Find the probability that T consists of two odd numbers and one even number.

Solution:

There are $C(20, 3)$ subsets of size three, and choosing any of them is equally likely. There are ten odd numbers and ten even numbers in S . A “success” means that we select two odd numbers from the ten odd numbers and one even number from the ten even numbers. Therefore

$$p(T \text{ has two odd numbers and one even number}) = \frac{C(10, 2) \cdot C(10, 1)}{C(20, 3)} \approx 0.395.$$

p.394, icon at Example 1

#8. Suppose $S = \{1, 2, \dots, 20\}$. You select a subset $T \subseteq S$ of size three. Find the probability that T consists of three prime numbers.

Solution:

There are $C(20, 3)$ subsets of size three, and choosing any of them is equally likely. There are eight primes in the first 20 integers — 2, 3, 5, 7, 11, 13, 17, 19. Therefore

$$p(T \text{ consists of three primes}) = \frac{C(8, 3)}{C(20, 3)} \approx 0.049.$$

p.394, icon at Example 1

#9. Suppose $S = \{1, 2, \dots, 20\}$. You select a subset $T \subseteq S$ of size three. Find the probability that the three numbers in T have a sum that is less than nine.

Solution:

There are $C(20, 3)$ subsets of size three, and choosing any of them is equally likely. There are four ways in which the three numbers can have a sum less than nine: 1, 2, 3; 1, 2, 4; 1, 2, 5; and 1, 3, 4. Therefore

$$p(\text{sum of the integers is less than nine}) = \frac{4}{C(20, 3)} \approx 0.004.$$

p.394, icon at Example 1

#10. A class has 20 women and 13 men. A committee of five is chosen at random. Find

- (a) p (the committee consists of all women).
- (b) p (the committee consists of all men)
- (c) p (the committee consists of all of the same sex)

Solution:

(a) There are $C(33, 5)$ possible committees of size five. Of these, there are $C(20, 5)$ committees consisting only of women. Therefore, the probability that the committee consists of all women is

$$\frac{C(20, 5)}{C(33, 5)} \approx 0.065.$$

(b) There are $C(33, 5)$ possible committees of size five. Of these, there are $C(13, 5)$ committees consisting only of men. Therefore, the probability that the committee consists of all women is

$$\frac{C(13, 5)}{C(33, 5)} \approx 0.005.$$

(c) There are $C(33, 5)$ possible committees of size five. Of these, there are $C(20, 5)$ committees consisting only of women and $C(13, 5)$ committees consisting only of men. Therefore, the probability that the committee consists of all women is

$$\frac{C(20, 5) + C(13, 5)}{C(33, 5)} \approx 0.07.$$

p.394, icon at Example 1

#11. What is the probability of getting more heads than tails, if you toss a fair coin

- (a) nine times?
- (b) ten times?

Solution:

(a) If you toss a fair coin nine times, half the time you will have more H than T and half the time you will have more T than H. Therefore, the probability that you obtain more heads than tails is $1/2$.

(b) If you toss a coin ten times, one of the following three possibilities must happen:

1. you toss equal numbers of H and T,
2. you toss more H than T,
3. you toss more T than H.

The sample space consists of all strings of ten letters, each of which is H or T. The number of ways of obtaining 5 H's and 5 T's is $C(10, 5)$. (To see this, note that we must choose a set of 5 of the 10 spots in a string for H's.)

Therefore, the number of ways of obtaining unequal numbers of heads and tails is $2^{10} - C(10, 5)$. Half of these will have more heads than tails, and half will have more tails than heads. Thus, the number of ways of obtaining more heads than tails is $\frac{1}{2}(2^{10} - C(10, 5)) = 2^9 - C(10, 5)/2$. Therefore, the probability of obtaining more heads than tails is

$$\frac{2^9 - C(10, 5)/2}{2^{10}} = \frac{1}{2} - \frac{C(10, 5)}{2^{11}}.$$

p.394, icon at Example 1

#12. A family has two children. They are not twins. You ring the doorbell of the house they live in and a girl answers the door. What is the probability that the other child in the family is a girl? Assume that in the births of two children the probability of the birth of a girl or a boy are independent events and that the probability of the birth of a child of either sex is $1/2$.

Solution:

The obvious answer is $1/2$, because according to the assumptions in this example the probability that any child is a girl is $1/2$. However this is not correct. The error here is in determining the sample space S . We know that the family cannot have two boys, because a girl answered the door. If we take as the sample space $\{1 \text{ girl and } 1 \text{ boy}, 2 \text{ girls}\}$, then the two outcomes are not equally likely. Having "1 girl and 1 boy" is twice as likely as having "2 girls". This is true because "1 girl and 1 boy" can occur in two ways: the older child is a girl and the younger child is a boy, or the older child is a boy and the younger child is a girl.

If we wish to use the basic definition of probability, $p(E) = |E|/|S|$, then the outcomes must all be equally likely. We can use the sample space where each element has the form "older child, younger child". In this case we begin with a sample space of size four: $\{GG, GB, BG, BB\}$. Note that the probability of having one child of each sex is $1/2$, not $1/4$. Given that the family has at least one girl, we eliminate the possibility BB to obtain sample space $S = \{GG, GB, BG\}$. Each outcome has probability $1/3$ and thus

$$p(\text{other child is a girl}) = 2/3.$$

Note: Suppose, in addition, we knew that it was the older child who answered the door. In this case the sample space becomes $S = \{GG, GB\}$ because the older child is not a boy. Thus, the probability that the other child is a girl is $1/2$.

Note: This example will be studied in the textbook from a slightly different point of view as Example 4 in Section 6.2.

p.397, icon at Example 9

#1. Suppose $S = \{1, 2, \dots, 20\}$. You select a subset $T \subseteq S$ of size three. Find the probability that T has at least one even number in it.

Solution:

There are $C(20, 3)$ subsets of size three, and choosing any of them is equally likely. It is easiest to use the rule $p(E) = 1 - p(\overline{E})$. Let E be the event “ T has at least one even number in it”. Therefore \overline{E} is the event “ T has only odd numbers in it”. We have

$$p(E) = 1 - p(\overline{E}) = 1 - \frac{C(10, 3)}{C(20, 3)} \approx 0.895.$$

p.397, icon at Example 9

#2. Suppose $S = \{1, 2, \dots, 20\}$. You select a subset $T \subseteq S$ of size three. Find the probability that T contains the numbers 10 or 20.

Solution:

There are $C(20, 3)$ subsets of size three, and choosing any of them is equally likely. We use the rule for finding the probability of the union of two events, where E is the event “ $10 \in T$ ” and F is the event “ $20 \in T$ ”. Note that we must subtract $p(E \cap F)$ because both 10 and 20 might be elements of T .

$$\begin{aligned} p(E \cup F) &= p(E) + p(F) - p(E \cap F) \\ &= \frac{C(19, 2)}{C(20, 3)} + \frac{C(19, 2)}{C(20, 3)} - \frac{C(18, 1)}{C(20, 3)} \\ &= \frac{C(19, 2) + C(19, 2) - C(18, 1)}{C(20, 3)} \approx 0.284. \end{aligned}$$

p.397, icon at Example 9

#3. A true/false quiz has 10 questions. If you randomly answer each question, what is the probability that you score at least 70%?

Solution:

To score at least 70%, you need to answer 7, 8, 9, or 10 questions correctly. There is $C(10, 10) = 1$ way to answer all ten questions correctly, $C(10, 9) = 10$ ways to correctly answer nine questions correctly,

$C(10, 8) = 45$ ways to answer eight questions correctly, and $C(10, 7) = 120$ ways to answer seven questions correctly. Thus, the probability of answering at least seven questions correctly is

$$\begin{aligned}
 p(\text{answer at least 7 correctly}) &= p(\text{answer 10 correctly}) + p(\text{answer 9 correctly}) + p(\text{answer 8 correctly}) \\
 &\quad + p(\text{answer 7 correctly}) \\
 &= \frac{1}{2^{10}} + \frac{10}{2^{10}} + \frac{45}{2^{10}} + \frac{120}{2^{10}} \\
 &= \frac{1 + 10 + 45 + 120}{2^{10}} \\
 &= \frac{176}{1024} \\
 &\approx 0.172.
 \end{aligned}$$

p.397, icon at Example 9

#4. In a lottery game, a winning set of five numbers is chosen from the set $\{1, 2, \dots, 44\}$. To play, you pick a set of five numbers. If your five numbers match the five winning numbers, you win first prize. If exactly four match, you win second prize. If exactly three match, you win third prize. If two or fewer numbers match, you win nothing. Find

- (a) $p(\text{win first prize})$.
- (b) $p(\text{win second prize})$.
- (c) $p(\text{win third prize})$.
- (d) $p(\text{win no prize})$.

Solution:

There are $C(44, 5)$ equally likely ways in which you can choose five numbers from the 44 numbers. Therefore, the sample space has size $C(44, 5)$.

(a) First prize can be won in only one way — match the five winning numbers. Therefore

$$p(\text{win first prize}) = \frac{C(5, 5)}{C(44, 5)} = \frac{1}{C(44, 5)} \approx 0.0000009.$$

$$(b) p(\text{win second prize}) = \frac{C(5, 4) \cdot C(39, 1)}{C(44, 5)} \approx 0.0002$$

$$(c) p(\text{win third prize}) = \frac{C(5, 3) \cdot C(39, 2)}{C(44, 5)} \approx 0.007$$

(d) Winning no prize is the same as not winning first, second, or third prize. Therefore

$$\begin{aligned}
 p(\text{win no prize}) &= 1 - p(\text{win first, second, or third prize}) \\
 &= 1 - \underbrace{\frac{C(5, 5)}{C(44, 5)}}_{\text{1st prize}} - \underbrace{\frac{C(5, 4) \cdot C(39, 1)}{C(44, 5)}}_{\text{2nd prize}} - \underbrace{\frac{C(5, 3) \cdot C(39, 2)}{C(44, 5)}}_{\text{3rd prize}} \\
 &\approx 0.993.
 \end{aligned}$$

p.397, icon at Example 9

#5. Several states play a lottery game call Mega Millions. A Mega Millions lottery game ticket has two halves: you pick a set of five numbers from the numbers 1 through 56 on the top half of the ticket, and one number (the Mega Ball number) from 1 through 46 on the bottom half of the ticket. (The Mega Ball number can be the same as one of the five numbers picked on the top half of the ticket.) Six winning numbers are chosen: five numbers from 1 through 56 and one Mega Ball number from the numbers 1 through 46. You win a prize if your choices match some or all of the winning numbers, as follows: five and Mega Ball, five and no Mega Ball, four and Mega Ball, four and no Mega Ball, three and Mega Ball, three and no Mega Ball, two and Mega Ball, one and Mega Ball, only the Mega Ball.

Suppose you purchase one ticket. Find the probability that you win each of these prizes.

Solution:

Elements of the sample space consist of all sets of five numbers chosen from $\{1, 2, 3, \dots, 56\}$, followed by one number from the set $\{1, 2, 3, \dots, 46\}$. For example, $\{3, 7, 17, 22, 49\}, 8$ is one element of the sample space. The number of elements in the sample space is $C(56, 5) \cdot C(46, 1) = 175,711,536$. We obtain the following probabilities. In each case the numerator is the number of ways of getting a “success” and the denominator is $C(56, 5) \cdot C(46, 1)$.

For example, if we wish to compute the probability of matching three numbers and not matching the Mega Ball, the number of ways of getting three matching numbers and no Mega Ball is counted as follows:

$$\overbrace{[C(5, 3) \ C(51, 2)]}^{5 \text{ numbers}} \cdot C(45, 1).$$

3 winning numbers
2 losing numbers
no Mega Ball

We must choose five numbers between 1 and 56 — three will match the five winning numbers and two will match the 51 losing numbers — and we must match one of the 45 losing Mega Ball numbers. We then divide by the number of possibilities, $C(56, 5) \cdot C(46, 1)$. The following are the probabilities and the approximate odds in favor of the event happening. (If E is an event and we take $p(E)$ and write it in the form $1/x$, the odds in favor of E are written as $1:x$.)

$$p(\text{five numbers and Mega Ball}) = \frac{C(5, 5) \cdot C(1, 1)}{C(56, 5) \cdot C(46, 1)} \approx 0.000000057 \quad (\text{odds: } 1:175,711,536)$$

$$p(\text{five numbers and no Mega Ball}) = \frac{C(5, 5) \cdot C(45, 1)}{C(56, 5) \cdot C(46, 1)} \approx 0.000000256 \quad (\text{odds: } 1:3,904,701)$$

$$p(\text{four numbers and Mega Ball}) = \frac{[C(5, 4) \cdot C(51, 1)] \cdot C(1, 1)}{C(56, 5) \cdot C(46, 1)} \approx 0.00000145 \quad (\text{odds: } 1:689,065)$$

$$p(\text{four numbers and no Mega Ball}) = \frac{[C(5, 4) \cdot C(51, 1)] \cdot C(45, 1)}{C(56, 5) \cdot C(46, 1)} \approx 0.0000653 \quad (\text{odds: } 1:15,313)$$

$$p(\text{three numbers and Mega Ball}) = \frac{[C(5, 3) \cdot C(51, 2)] \cdot C(1, 1)}{C(56, 5) \cdot C(46, 1)} \approx 0.00007256 \quad (\text{odds: } 1:13,781)$$

$$p(\text{two numbers and Mega Ball}) = \frac{[C(5, 2) \cdot C(51, 3)] \cdot C(1, 1)}{C(56, 5) \cdot C(46, 1)} \approx 0.0012 \quad (\text{odds: } 1:844)$$

$$p(\text{three numbers and no Mega Ball}) = \frac{[C(5, 3) \cdot C(51, 2)] \cdot C(45, 1)}{C(56, 5) \cdot C(46, 1)} \approx 0.0033 \quad (\text{odds: } 1:306)$$

$$p(\text{one number and Mega Ball}) = \frac{[C(5, 1) \cdot C(51, 4)] \cdot C(1, 1)}{C(56, 5) \cdot C(46, 1)} \approx 0.007 \quad (\text{odds: } 1:141)$$

$$p(\text{no numbers and Mega Ball}) = \frac{C(51, 5) \cdot C(1, 1)}{C(56, 5) \cdot C(46, 1)} \approx 0.0134 \quad (\text{odds: } 1:75)$$

We can also find the probability that we win a prize by taking the sum of all the probabilities we found. We obtain $p(\text{ticket wins a prize}) \approx 0.025$ (approximate odds 1:40). Hence, $p(\text{win no prize}) \approx 1 - .025 = 0.975$ (approximate odds 39:40).

p.397, icon at Example 9

#6. Six cards, numbered 1, 2, 3, 4, 5, 6, are placed in a row. Let E be the event that 1 and 2 are next to each other or 3 and 4 are next to each other. Which is more likely: E or \bar{E} ?

Solution:

The six cards can be put in a row in $6! = 720$ ways. We need to determine the size of event E . To do this, we write $E = F \cup G$, where F is the set of arrangements with 1 and 2 adjacent and G is the set of arrangements with 3 and 4 adjacent.

To find $|F|$, temporarily glue cards 1 and 2 together, giving five cards: 12, 3, 4, 5, 6. These five cards can be put in a row in $5! = 120$ ways. For each of the 120 patterns, the “12” card can be separated, with 1 on the left of 2, or 2 on the left of 1. Therefore, $|F| = 2 \cdot 120 = 240$.

Likewise, if we count the number of arrangements with 3 and 4 adjacent, we obtain $|G| = 2 \cdot 120 = 240$.

Because $|E| = |F \cup G| = |F| + |G| - |F \cap G|$ by the inclusion-exclusion principle, we need to find $|F \cap G|$. To do this, glue cards 1 and 2 together and glue cards 3 and 4 together. This gives us four cards: 12, 34, 5, 6, which we can arrange in $4! = 24$ ways. We can interchange the positions of 1 and 2 and interchange the positions of 3 and 4 in two ways each. Therefore, $|F \cap G| = 24 \cdot 2 \cdot 2 = 96$.

Therefore,

$$\begin{aligned} p(E) &= p(F \cup G) \\ &= p(F) + p(G) - p(F \cap G) \\ &= \frac{240}{720} + \frac{240}{720} - \frac{96}{720} \\ &= \frac{384}{720}. \end{aligned}$$

Therefore $p(\bar{E}) = 1 - \frac{384}{720} = \frac{336}{720}$. This tells us that event E is more likely.

p.397, icon at Example 9

#7. You pick five numbers, without replacement, from the set $\{1, 2, 3, \dots, 24, 25\}$. What is the probability that the sum of the numbers chosen is odd?

Solution:

We can think of the experiment of choosing five numbers in two ways: pick all five numbers at once (i.e., order does not matter), or pick the five numbers in succession (i.e., order matters).

Suppose we interpret the problem as one where we choose the five numbers all at once. Then the sample space S consists of all subsets of five numbers chosen from the set $\{1, 2, 3, \dots, 24, 25\}$, and we have $|S| = C(25, 5)$.

The only way to have the sum of the five numbers chosen be odd is to have one, three, or five odd numbers in the set of five chosen. Let E be the event that one odd number is chosen, F the event that three odd

numbers are chosen, and G the event that five odd numbers are chosen. We need to find $p(E \cup F \cup G)$. But E , F , and G are disjoint, so $p(E \cup F \cup G) = p(E) + p(F) + p(G)$. Using the fact that there are 13 odd numbers and 12 even numbers, we have

$$\begin{aligned} p(E \cup F \cup G) &= p(E) + p(F) + p(G) \\ &= \frac{C(13,1) \cdot C(12,4)}{C(25,5)} + \frac{C(13,3) \cdot C(12,2)}{C(25,5)} + \frac{C(13,5) \cdot C(12,0)}{C(25,5)} \\ &= \frac{C(13,1) \cdot C(12,4) + C(13,3) \cdot C(12,2) + C(13,5) \cdot C(12,0)}{C(25,5)} \\ &\approx 0.501. \end{aligned}$$

Suppose we solve the problem again, but we assume that the order in which the five numbers are chosen matters. In this case the sample space S consists of all sequences of length five of numbers from 1 to 25, and we have $|S| = P(25, 5)$.

We need to compute the number of successes. In order to have an odd sum, we need to compute the number of sequences of length five consisting of one, three, or five odd integers. To compute the number of sequences of length five with exactly one odd integer, we can take a row of five blanks and choose one of them to hold the one odd number chosen. The choice of one blank can be done in $C(5, 1)$ ways and the number of ways to insert an odd integer in this blank is $P(13, 1)$. Because the number of ways to pick four even integers to place in the other four blanks is $P(12, 4)$, the number of ways to obtain a sequence of length five with exactly one odd integer is $C(5, 1) \cdot P(13, 1) \cdot P(12, 4)$. Therefore, the probability of obtaining a sequence of length five with exactly one odd integer is

$$[C(5, 1) \cdot P(13, 1) \cdot P(12, 4)]/P(25, 5).$$

Similarly, the number of ways to obtain a sequence of length five with exactly three odd integers is

$$[C(5, 3) \cdot P(13, 3) \cdot P(12, 2)]/P(25, 5)$$

and the number of ways to obtain a sequence of length five with all five odd integers is

$$[C(5, 5) \cdot P(13, 5)]/P(25, 5).$$

Therefore we have

$$\begin{aligned} p(E \cup F \cup G) &= p(E) + p(F) + p(G) \\ &= \frac{C(5, 1) \cdot P(13, 1) \cdot P(12, 4)}{P(25, 5)} + \frac{C(5, 3) \cdot P(13, 3) \cdot P(12, 2)}{P(25, 5)} + \frac{C(5, 5) \cdot P(13, 5)}{P(25, 5)} \\ &= \frac{C(5, 1) \cdot P(13, 1) \cdot P(12, 4) + C(5, 3) \cdot P(13, 3) \cdot P(12, 2) + C(5, 5) \cdot P(13, 5)}{P(25, 5)} \\ &\approx 0.501. \end{aligned}$$

Note: In this case the same answer is obtained regardless of whether or not we assume order matters.