Show All Solutions

Rosen, Discrete Mathematics and Its Applications, 6th edition
Extra Examples

Section 1.7—Proof Methods and Strategy

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

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p.87, icon at Example 1

#1. Prove that there is only one pair of positive integers that is a solution to $3x^2 + 2y^2 = 30$.

See Solution

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p.88, icon at Example 3

#1. Prove that the square of every even integer ends in 0, 4, or 6.

See Solution

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p.88, icon at Example 3

#2. Prove that the following is true for all real numbers x and y: $\max(x, y) = \frac{1}{2}(x + y + |x - y|)$.

See Solution

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p.88, icon at Example 3

#3. Prove that the square of every odd integer ends in 1, 5, or 9.

See Solution

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p.91, icon at Example 10

#1. Prove that there are numbers x and y whose sum is 5 and whose product is 2. (Note that we are only required to show that x and y exist; we are not required to find specific values for x and y.)

See Solution

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p.93, icon at Example 13

#1. Show that if x is a nonzero rational number, then there is a unique rational number y such that $xy = 2$.

See Solution

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p.95, icon at Example 14

#1. Prove that the square of every odd integer has the form $8k + 1$, where k is an integer.

See Solution

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p.95, icon at Example 14

#2. Suppose a and b are integers such that $2a = b^2 + 3$. Prove that a is the sum of three squares.

See Solution

p.96, icon at Example 16

#1. Over the centuries, mathematicians have tried to adapt the proofs of others to obtain new results. A classic example of this is the proof of the Four Color Theorem. The Four Color Theorem states that the countries on every map can be colored with at most four colors so that two countries that share a common border have different colors.

In the nineteenth century it was proved that five colors are sufficient to color the countries on any map so that countries that share a common border receive different colors. No one was able to produce a map that required five colors, but no one was able to prove that four colors were sufficient to color every possible map. In 1976, two mathematicians, Kenneth Appel and Wolfgang Haken, were able to prove that four colors suffice to color the countries of every map so no countries that share a common border have the same color. This proof, complicated and very lengthy, was an adaptation of the much simpler proof that five colors always suffice. Map coloring problems will be discussed in detail in Section 9.8.

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p.96, icon at Example 16

#2. (Adapted from Problem A4 from the 1988 William Lowell Putnam Mathematics Competition.)

(a) Suppose that every point of the plane is painted one of two colors, a or b. Must there be two points of the same color that are exactly one inch apart?

(b) Suppose that every point of the plane is painted one of three colors, a, b, or c. Must there be two points of the same color that are exactly one inch apart?

(c) Prove that if nine colors are allowed, a coloring is possible with the property that no two points one inch apart have the same color.

See Solution

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p.96, icon at Example 17

#1. Prove or disprove the following conjecture: For all real numbers x, $x^2 > x$.

See Solution

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p.96, icon at Example 17

#2. Prove or disprove the statement "The square of every integer ends in $0, 1, 4, 5$, or 9."

See Solution

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p.96, icon at Example 17

#3. Prove or disprove the statement "There is a number x such that $x > x + 2$."

See Solution

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p.97, icon below start of "Proof Strategy in Action" subsection

#1. Suppose that a, b, c are real numbers and each is less than the sum of the other two. Prove that all three numbers are positive.

See Solution

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p.97, icon below start of "Proof Strategy in Action" subsection

#2. Suppose N is the sum of squares of two integers. Prove that $2N$ is also the sum of squares of two integers.

See Solution

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p.97, icon below start of "Proof Strategy in Action" subsection

#3. Suppose $n > 1$ is not prime. (A *prime* number is a positive integer greater than 1 that has no positive integer divisors other than 1 and itself.) Are there positive integers x, y, and z such that $n = xy + yz + zx + 1$?

See Solution

p.98, icon at Example 19

#1. A rectangular floor is tiled using two kinds of tiles, square 2×2 tiles and rectangular 1×4 tiles. Suppose that one tile is destroyed, but that one tile of the other kind is available. It is possible to tile the entire floor, using the original tiles with this one replacement, by rearranging the tiles?

See Solution

p.98, icon at Example 19

#2. Can you tile a 17×28 checkerboard using 4×7 tiles?

See Solution

p.98, icon at Example 19

#3. An *T-tetromino* consists of a row of three squares with a fourth square directly above the middle square. Show that a 14×14 checkerboard cannot be tiled with T-tetrominoes.

See Solution

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p.98, icon at Example 19

#4. Show that in any tiling of an 8×8 checkerboard by tetrominoes, where any of the five different kinds of tetrominoes can be used, the number of T-tetrominos must be even.

See Solution

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p.98, icon at Example 19

#5. In a tiling of a checkerboard by dominoes, a *fault line* is a vertical or horizontal line that cuts the checkerboard into two pieces without passing through any of the dominoes. Show that whenever a 6×6 checkerboard is tiled with dominoes, the tiling has a fault line. That is, no matter how the 6×6 checkerboard is tiled with dominoes, it is possible to cut the checkerboard in two without passing through one of the dominoes.

See Solution

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p.98, icon at Example 19

#6. Prove or disprove that there is a tiling of the 5×6 checkerboard that does not have a fault line, that is, a vertical or horizontal line that cuts the checkerboard in two without passing through one of the dominoes.

See Solution

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p.98, icon at Example 19

#7. How many different pentominoes, that is, arrangements of five squares of a checkerboard joined along edges, are there, where two such arrangements are considered the same if one can be obtained from the other by a rotation or a flipping?

See Solution
